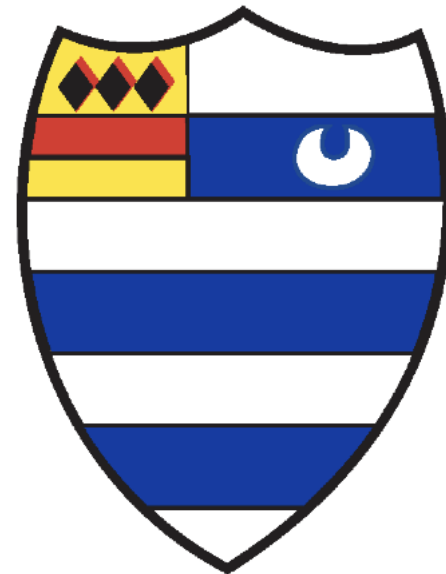


# Stanford Junior and Infant School Calculation Policy

This policy has been adapted from the White Rose Calculation Policy, based on a Mastery approach. The focus is on deepening conceptual understanding. Problems are modelled with **concrete** materials, then using **pictorial** representations of the materials, leading to use of **abstract** symbols.



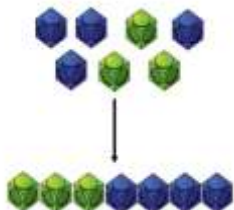
The indications of Year groups are for guidance only; teachers use their professional judgement as to whether consolidation of a certain concept is required before moving to the next.

# Addition

Early Years-Year 1

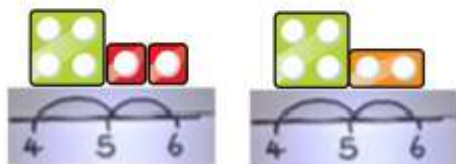
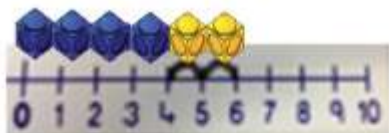
## Concrete

Combining two parts to make a whole (variety of resources e.g. eggs, shells, teddy bears, cars).



Year 1

Starting with the bigger number and counting on using number lines using cubes or Numicon.



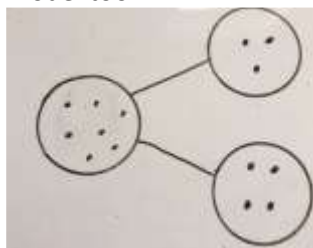
Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$

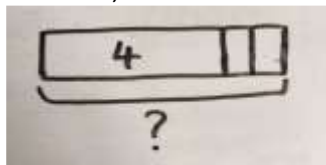


## Pictorial

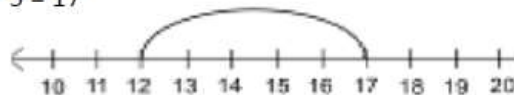
Children to represent the cubes using dots or crosses. They could put each part on a part-whole model too.



A bar model which encourages the children to count on, rather than count all.

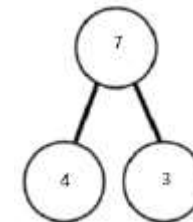


Start at the larger number and jump on in ones or one jump  
 $12 + 5 = 17$



## Abstract

$4 + 3 = 7$  or  $7 = 4 + 3$   
 Four is a part, 3 is a part and the whole is seven.



The abstract number line:  
 What is 2 more than 4?  
 What is the sum of 2 and 4?  
 What is the total of 4 and 2?  
 $4 + 2$



Put the larger

number in your head and count on the smaller number.

Children to develop an understanding of equality, eg

$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

# Addition

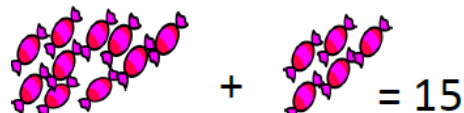
Year 2

**Adding 3 single digits**



Combine to make 10 if possible; or bridge 10, then add the third digit.

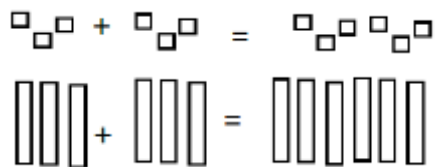
Children regroup and draw representations



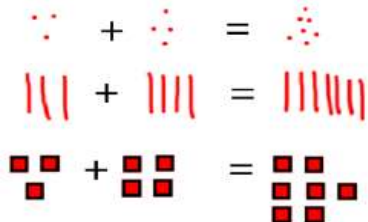
Combine to make 10 if possible; or bridge 10, then add the third digit.

$$\begin{matrix} 4 + 7 + 6 = 10 + 7 \\ 10 \\ = 17 \end{matrix}$$

**Using known facts**



Children draw representations of H,T,U



I know  $3 + 3 = 6$   
so  $30 + 30 = 60$   
and  $300 + 300 = 600$

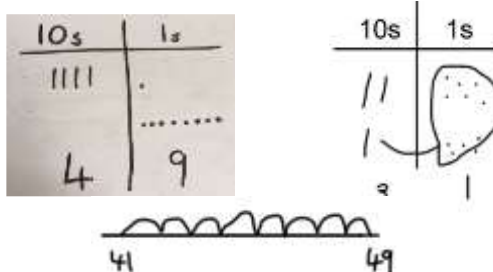
**TU + U**

Use dienes to continue to develop understanding of place value e.g.  $41 + 8$

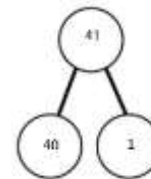


Initially not crossing 10.  
Then moving on to regrouping (exchanging) when crossing 10 e.g.  $26 + 5$

Children to represent the dienes e.g. lines for tens and dots for units.



$$41 + 1 + 8 =$$



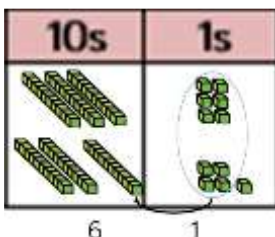
$$\begin{matrix} 8 \\ 9 \\ 40 + 9 = 49 \end{matrix}$$

	4	1
+		8
		9
4	9	

	2	6
+		5
		1
2	6	1

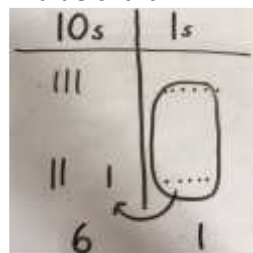
**TU + TU**

Use dienes to continue to develop understanding of partitioning and place value e.g.  $36 + 25$

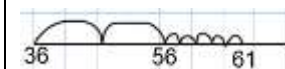
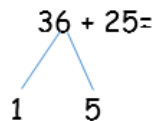


Number lines may also be used.

Children to represent the base 10 in a place value chart.



$$\begin{aligned} 36 + 25 &= 30 + 20 + 5 + 5 + 1 \\ &= 50 + 10 + 1 \\ &= 61 \end{aligned}$$



Formal methods:

	3	6
+	2	5
		1
5	0	
6	1	

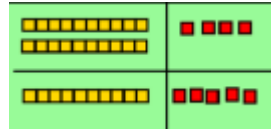
	36
+	25
1	
61	

# Addition

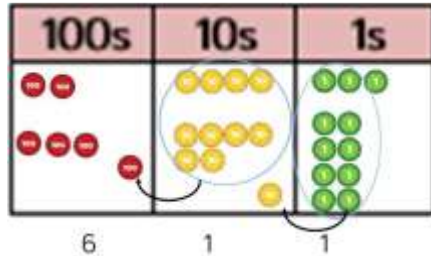
Year 3

**HTU + TU, HTU + HTU**

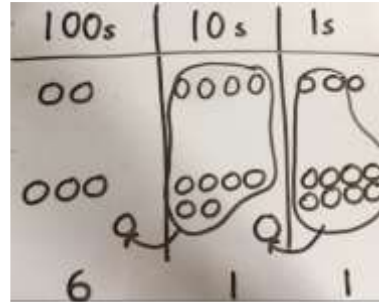
Model with Dienes  
Move on to place value  
counters.



When there are 10 ones in the 1s column,  
exchange for 1 ten; when there are 10 tens in the  
10s column, exchange for 1 hundred.



Children to represent the counters in a place value  
chart, circling when they make an exchange.



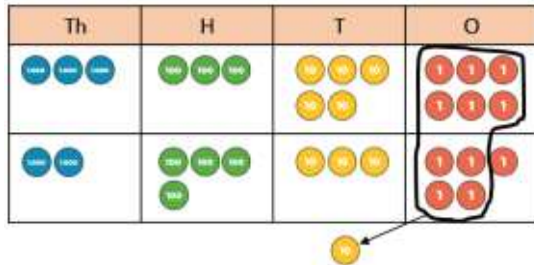
Written method

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1 \quad 1 \end{array}$$

Year 4

**ThHTU**

Using place value counters up to thousands.



As above, including thousands column

	Th	H	T	O
	3	3	5	6
+	2	4	3	5
	5	7	9	1
				1

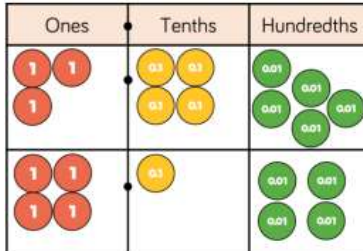
Relate to money and measures problems

Year 5

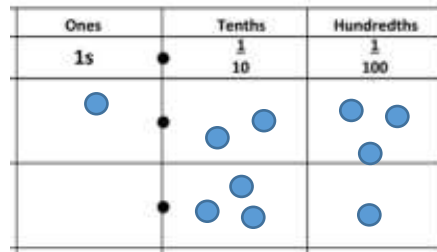
**TTh Th H T U and U. t h**

Working beyond 4 digits

Adding decimals with 2 decimal places



Children represent the counters in a decimal  
place value chart



1.23

0.31

$$\begin{array}{r} 4.55 \\ + 3.07 \\ \hline \hline \end{array}$$

# Addition

Year 6

**Add several numbers of increasing complexity**

Using place value counters as Year 5

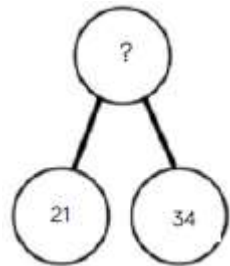
Adding money and measures, including decimals with different numbers of decimal places

As Year 5



Use zero as a place holder

## Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:  
In year 3, there are 21 children and in year 4, there are 34 children.  
How many children in total?

$21 + 34 = 55$ . Prove it

$$21 + 34 = \underline{\quad}$$

$$\underline{\quad} = 21 + 34$$

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

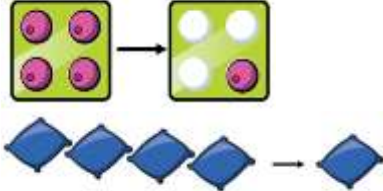
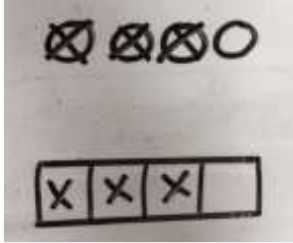
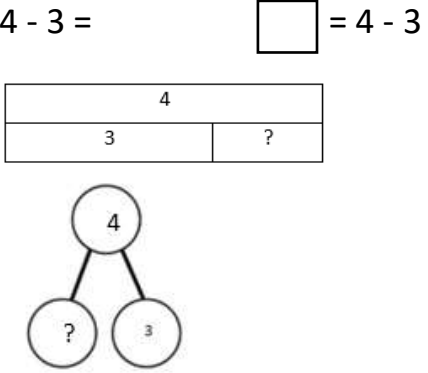

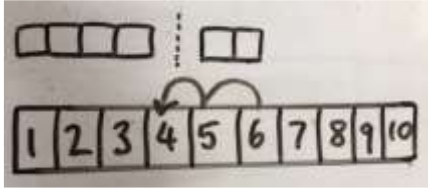
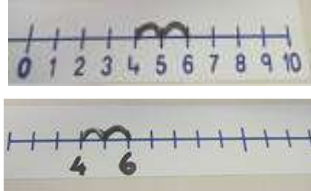

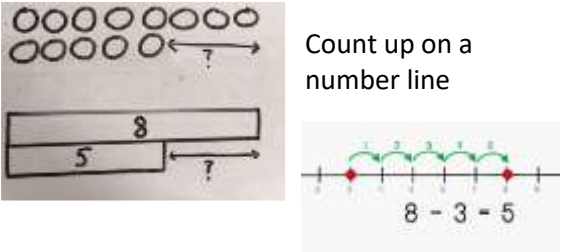
Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
● ●	●
● ● ●	?
?	5

# Subtraction

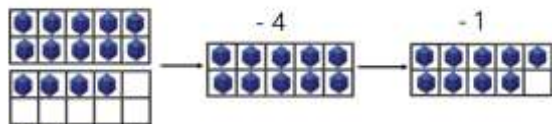
		Concrete	Pictorial	Abstract
Subtraction	Early Years-Year 1	<p><b>Taking away ones</b> Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). <math>4 - 3 = 1</math></p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p><math>4 - 3 =</math> <input type="text"/> <math>= 4 - 3</math></p> 
	Year 1	<p><b>Counting back</b> Using number lines or number tracks, children start with 6 and count back 2. <math>6 - 2 = 4</math></p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Begin to use an empty number line.</p>  <p>Mental: Put 13 in your head, count back 4. What number do you stop on?</p>
		<p><b>Finding the difference</b> Using cubes, Numicon or Cuisenaire rods, other objects can also be used. Calculate the difference between 8 and 5.</p> 	<p>Children to draw the objects they have used or use a bar model to illustrate what they need to calculate.</p>  <p>Count up on a number line</p>	<p>Find the difference between 8 and 5. <math>8 - 5</math>, the difference is _____</p> <p>Same difference: <math>9 - 6 = 8 - 5</math></p> <p>Word problems: Hannah has 12 sweets, her sister has 5. How many more does Hannah have?</p>

# Subtraction

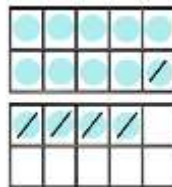
Year 1

## Making 10 using ten frames

$14 - 5$



Children present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

$$14 - 5 = 14 - 4 - 1$$

$$= 10 - 1$$

$$= 9$$

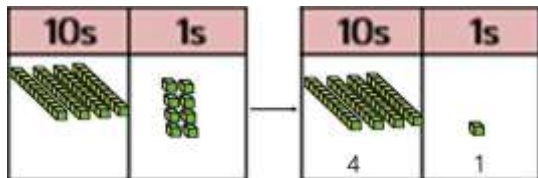
How many do we take off first to get to 10?  
How many are left to take off?

Year 2

## TU-U

Column method using dienes.

$48 - 7$

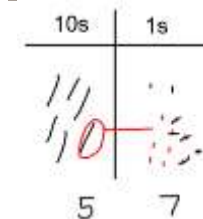
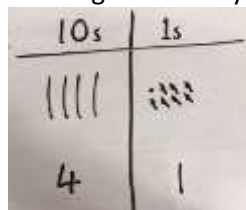


Initially not crossing 10.

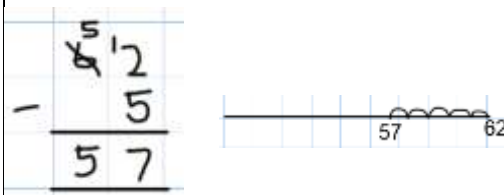
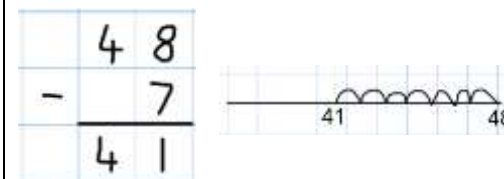
Then moving on to regrouping (exchanging) when crossing 10 e.g.  $62 - 5$

Number lines may also be used.

Children to represent the base 10 pictorially, crossing out as they take away.



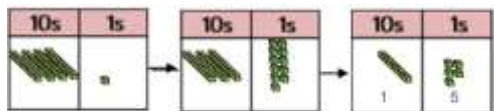
Column method or children could count back 7.



## TU-TU

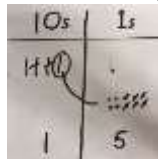
Column method using base 10 and having to exchange, using dienes.

$41 - 26$



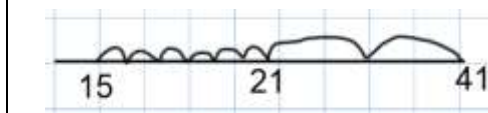
Number lines may also be used.

Represent the base 10 pictorially, remembering to show the exchange.



Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because  $41 = 30 + 11$ .

$$\begin{array}{r} 3 \text{ } 1 \\ - 26 \\ \hline 15 \end{array}$$



# Subtraction

Year 3

**HTU-TU: no regrouping**

Model with Dienes



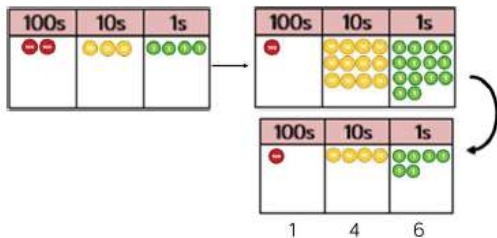
Draw representation as Year 2

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

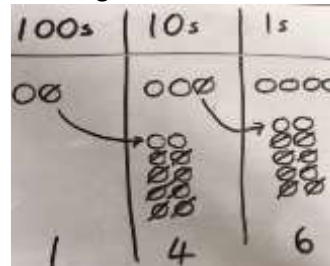
**HTU-TU: with regrouping**

Column method using place value counters to exchange.

$$234 - 88$$



Represent the place value counters pictorially; remembering to show what has been exchanged.



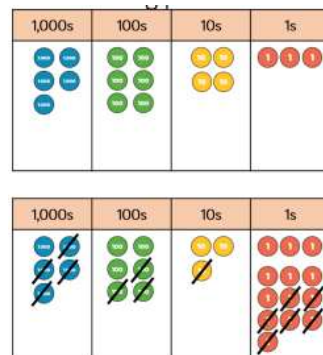
Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} \overset{2}{2} \overset{1}{3} 4 \\ - 88 \\ \hline 6 \end{array}$$

Year 4

**Subtracting tens and ones**

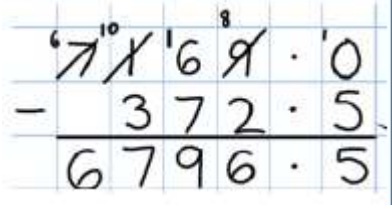
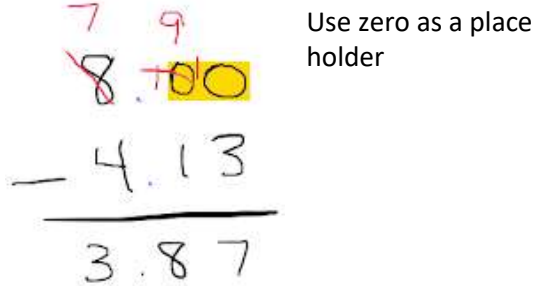
Up to 4 digits.



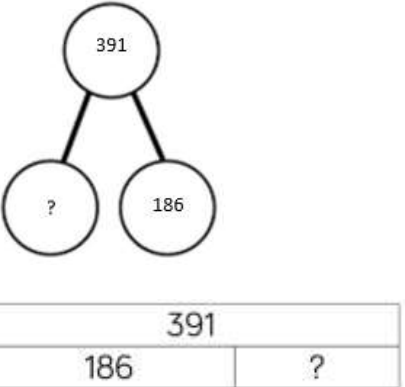
Represent the place value counters as above; remembering to show what has been exchanged.

	Th	H	T	O
	5	6	<del>3</del> 4	13
-	4	3	1	6
	1	3	2	7



	Year 5	<b>Subtracting numbers with at least 4 digits</b> Extending place value chart from Year 4 Introducing decimal subtraction with money and measures calculations.	Represent the place value counters as above; remembering to show what has been exchanged.	Line up the decimal point; use zero for place holder 
	Year 6	<b>Subtracting increasingly complex numbers and decimals</b>		

**Conceptual variation; different ways to ask children to solve 391 - 186**

	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>Calculate the difference between 391 and 186.</p>	<p>___ = 391 - 186</p> $\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$ <p>What is 186 less than 391?</p>	<p>Missing digit calculations.</p> $\begin{array}{r} 39\Box \\ -\Box\Box6 \\ \hline \Box05 \end{array}$
--	--	---	---

# Multiplication

Year 1

## Concrete

**Repeated grouping/repeated addition**

$$4 \times 3$$

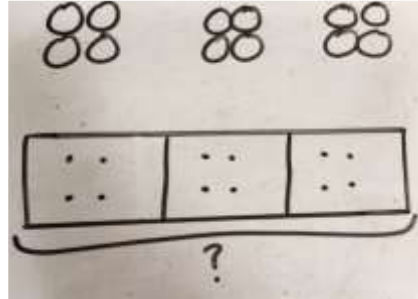
$$4 + 4 + 4$$

There are 4 in 3 equal groups.



## Pictorial

Children to represent the practical resources in a picture and use a bar model.



## Abstract

Written calculations

$$4 \times 3 = 12$$

$$12 = 4 \times 3$$

$$4 + 4 + 4 = 12$$

$$12 = 4 + 4 + 4$$

**Doubling**

Use a variety of resources including cubes and Numicon

Double 4:



Children draw pictures of doubles



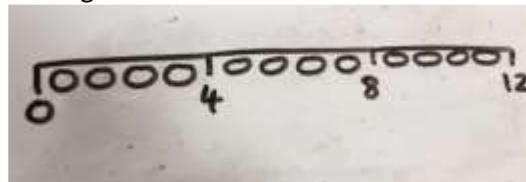
$$2 \times 4 = 8$$

Number lines to show repeated groups-

$$4 \times 3$$



Represent this pictorially alongside a number line e.g.



Abstract number line showing three jumps of four.

$$4 \times 3 = 12$$



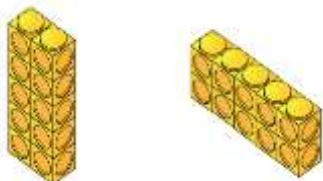
# Multiplication

Year 2

## Use arrays to illustrate commutativity

Counters and other objects can also be used.

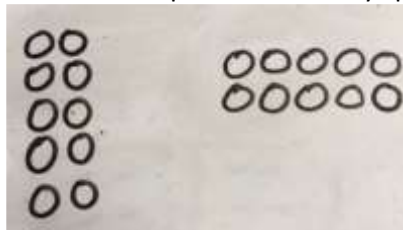
$$2 \times 5 = 5 \times 2$$



2 lots of 5

5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$2 \times 5 = 10 \quad \text{or} \quad 10 = 2 \times 5$$

$$5 \times 2 = 10 \quad \text{or} \quad 10 = 5 \times 2$$

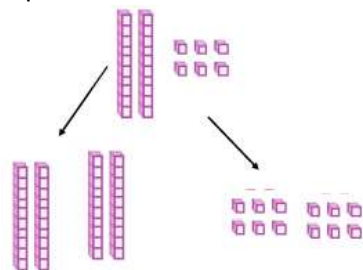
$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

## Doubling

Use Dienes and place value counters

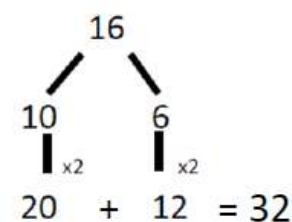
Double 26:



Children draw representations of materials

Doubling 2-digit numbers:

Partition and double each part, recombine

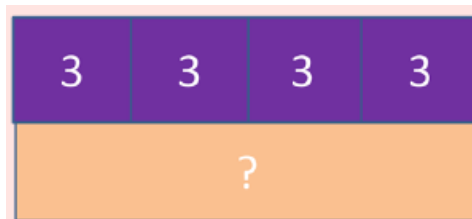
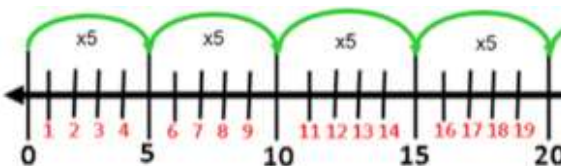


## Counting in Multiples of 2, 3, 4, 5, 10

Groups of objects/fingers used for skip counting



Number lines, counting sticks, bar models

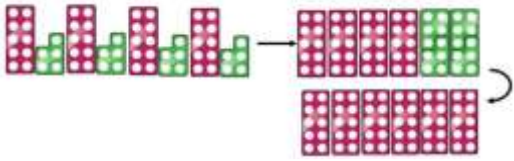
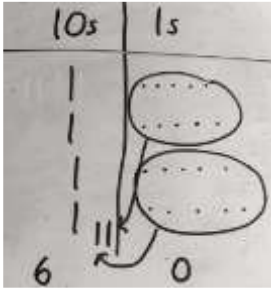
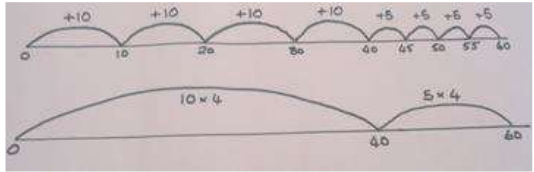
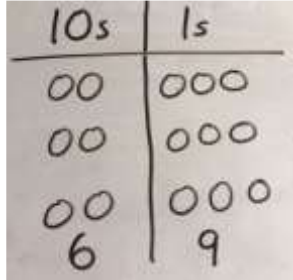
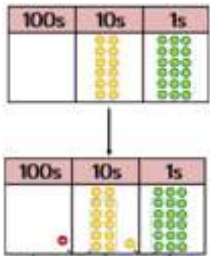
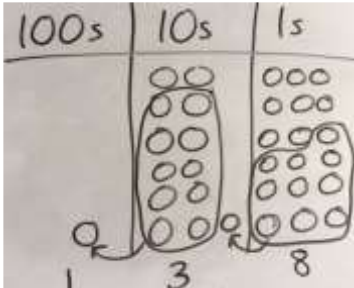


Count in 2s etc verbally

Write sequences of multiples

0, 2, 4, 6, 8..

0, 5, 10, 15,...

	Year 3	<p><b>U x TU: Partitioning</b> Partition to multiply using Numicon or dienes <math>4 \times 15</math></p>  <p>Relating to place value chart</p> <table border="1" data-bbox="371 478 622 651"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>■ ■ ■ ■</td> <td>■ ■ ■ ■ ■ ■ ■ ■</td> </tr> <tr> <td>■ ■ ■ ■</td> <td>■ ■ ■ ■ ■ ■ ■ ■</td> </tr> <tr> <td>■ ■ ■ ■</td> <td>■ ■ ■ ■ ■ ■ ■ ■</td> </tr> </tbody> </table> <p style="text-align: right;"><math>3 \times 26</math></p>	Tens	Ones	■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■	■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■	■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■	<p>Children to represent the concrete manipulatives pictorially.</p>  <p style="text-align: right;"><math>4 \times 15</math></p>	<p>Children to be encouraged to show the steps they have taken.</p> $  \begin{array}{r}  4 \times 15 \\  \swarrow \searrow \\  10 \quad 5 \\  \\  10 \times 4 = 40 \\  5 \times 4 = 20 \\  40 + 20 = 60  \end{array}  $ <p>A number line can also be used</p> 
Tens	Ones											
■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■											
■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■											
■ ■ ■ ■	■ ■ ■ ■ ■ ■ ■ ■											
	Year 4	<p><b>U x TU: Formal column method</b> Place value counters or dienes. No exchanging. <math>3 \times 23</math></p> <table border="1" data-bbox="371 786 607 986"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>● ● ● ●</td> <td>● ● ● ● ● ● ● ●</td> </tr> <tr> <td>● ● ● ●</td> <td>● ● ● ● ● ● ● ●</td> </tr> <tr> <td>● ● ● ●</td> <td>● ● ● ● ● ● ● ●</td> </tr> </tbody> </table> <p style="text-align: center;">6      9</p>	10s	1s	● ● ● ●	● ● ● ● ● ● ● ●	● ● ● ●	● ● ● ● ● ● ● ●	● ● ● ●	● ● ● ● ● ● ● ●	<p>Children to represent the counters pictorially.</p> 	<p>Children to record what it is they are doing to show understanding.</p> $  \begin{array}{r}  3 \times 23 \\  \swarrow \searrow \\  20 \quad 3 \\  \\  3 \times 20 = 60 \\  3 \times 3 = 9 \\  60 + 9 = 69  \end{array}  \qquad  \begin{array}{r}  23 \\  \times 3 \\  \hline  69  \end{array}  $
10s	1s											
● ● ● ●	● ● ● ● ● ● ● ●											
● ● ● ●	● ● ● ● ● ● ● ●											
● ● ● ●	● ● ● ● ● ● ● ●											
		<p><b>U x TU: Formal column method</b> Formal column method with place value counters. With an exchange, crossing 10.</p>  <p style="text-align: right;"><math>6 \times 23</math></p>	<p>Children to represent the counters/base 10, pictorially e.g. the image below.</p>  <p style="text-align: right;"><math>23 \times 6</math>:</p>	<p>Formal written method</p> $  \begin{array}{r}  6 \times 23 = \\  23 \\  \times 6 \\  \hline  138 \\  \hline  11  \end{array}  $								

# Multiplication

Year 5

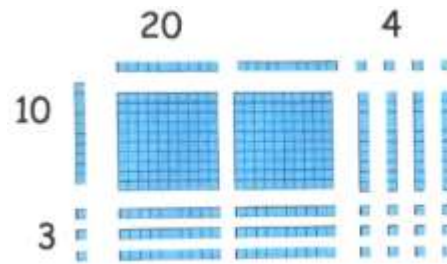
## Grid method

Place value counters or dienes, partitioning numbers, multiplying separate parts, then combining.



14 x 13

## Representations of Dienes



## Recording grid, with addition alongside

x	300	20	7
4	1200	80	28

327

x 4  
28

Leading to compact method

80

	3	2	7
x			4
	1	3	0
		2	8

1200

1308

Year 5-6

## Column multiplication up to 4 digits x 2 digits and 3 digits x 3 digits

Children must be confident with the abstract calculations HTU x U and HTU x T  
Concrete materials can still be used.

Pictorial representations as above

Formal written method

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 11
 \end{array}$$

Answer: 3224

Year 6

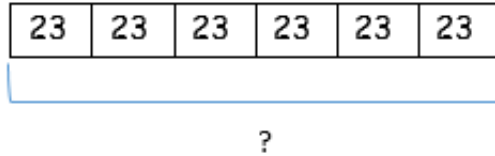
## Multiplication of decimals

Formal written method

$$\begin{array}{r}
 0.26 \\
 \times 8 \\
 \hline
 2.08
 \end{array}$$

Ensure the answer has the same number of decimal places as the question

# Conceptual variation; different ways to ask children to solve $6 \times 23$



Mai had to swim 23 lengths, 6 times a week.  
How many lengths did she swim in one week?

With the counters, prove that  $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\underline{\quad} = 6 \times 23$$

$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array}$	$\begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$
---	---

What is the calculation?

What is the product?

100s	10s	1s																							
	<table style="margin: 0 auto;"> <tr><td>●</td></tr> <tr><td>●</td></tr> <tr><td>●</td></tr> <tr><td>●</td></tr> <tr><td>●</td></tr> <tr><td>●</td></tr> <tr><td>●</td></tr> </table>	●	●	●	●	●	●	●	<table style="margin: 0 auto;"> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td></tr> </table>	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●
●																									
●																									
●																									
●																									
●																									
●																									
●																									
●	●																								
●	●																								
●	●																								
●	●																								
●	●																								
●	●																								
●	●																								
●	●																								

# Division

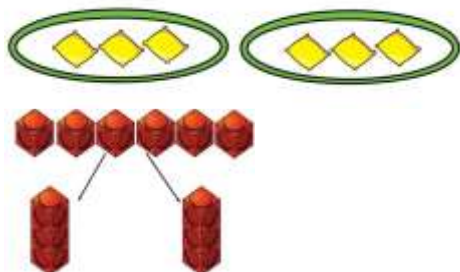
## Concrete

## Pictorial

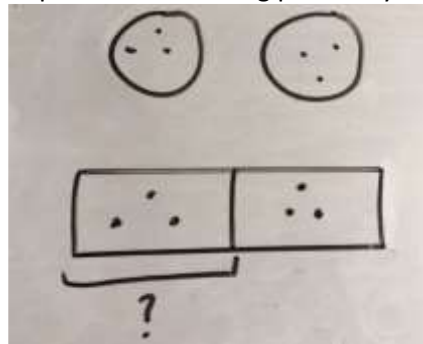
## Abstract

Year 1

**Division as Sharing**  
Using a range of objects.  
 $6 \div 2$



Represent the sharing pictorially.



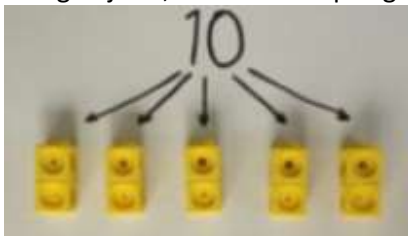
$6 \div 2 = 3$



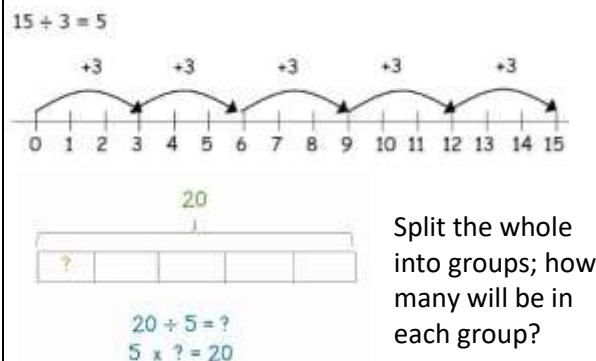
Children should also be encouraged to use their 2 times tables facts.

Year 2

**Division as Grouping**  
Using objects, divide into equal groups



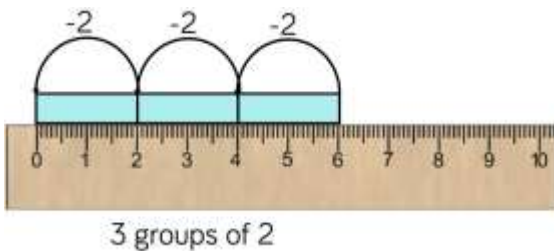
Using number lines and bar models



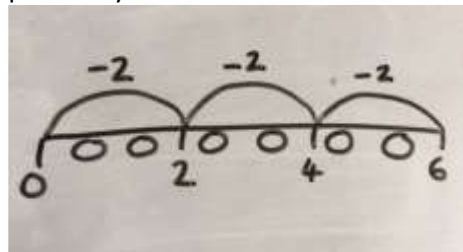
$20 \div 5 = 4$

Divide 20 into 5 groups. How many are in each group?

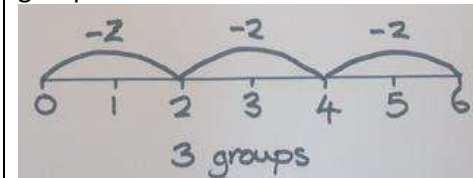
**Repeated subtraction** using Cuisenaire rods above a ruler.  $6 \div 2$



Children to represent repeated subtraction pictorially.



Abstract number line to represent the equal groups that have been subtracted.

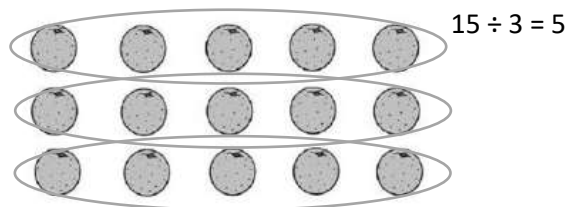
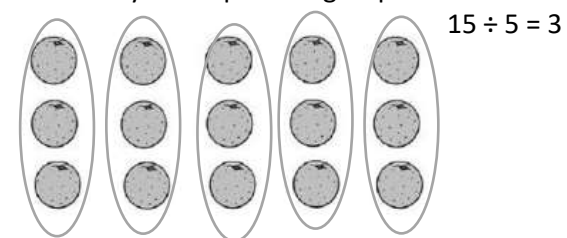


**Division with Arrays**

Link division with multiplication by creating an array and finding the related number sentences



Draw arrays and split into groups



Find the linking number sentences

- $15 \div 5 = 3$
- $15 \div 3 = 5$
- $5 \times 3 = 15$
- $3 \times 5 = 15$

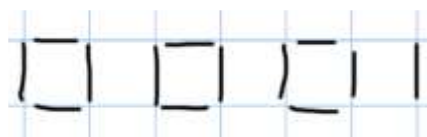
**TU ÷ U with remainders**

Using objects or lollipop sticks. Cuisenaire rods, above a ruler can also be used.



Use of lollipop sticks to form wholes- squares are made because we are dividing by 4. There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.



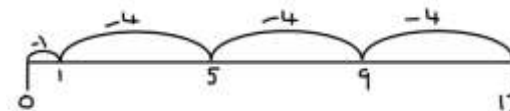
There are 3 whole squares, with 1 left over.

Draw dots and group them, clearly showing the remainder



Written:  $13 \div 4 = 3$  remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over'



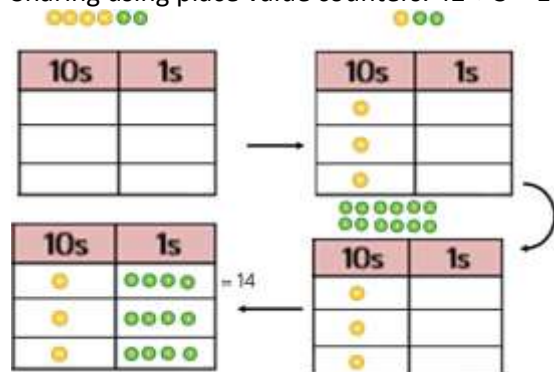


# Division

Year 4-5

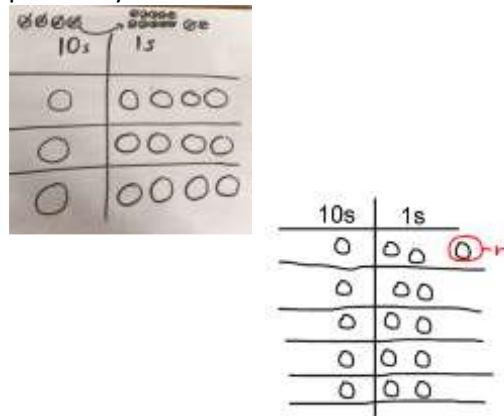
## TU ÷ U

Sharing using place value counters.  $42 \div 3 = 14$



Repeat with remainders e.g.  $61 \div 5 =$

Children to represent the place value counters pictorially.



Record in short division 'bus stop' format

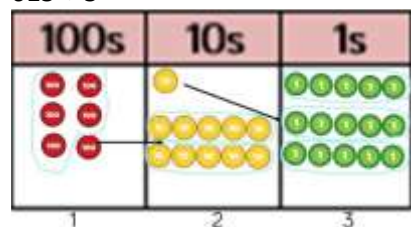
$$61 \div 5$$

$$\begin{array}{r} 12 \text{ r } 1 \\ 5 \overline{) 61} \end{array}$$

## HTU ÷ U

Short division using place value counters to group.

$$615 \div 5$$

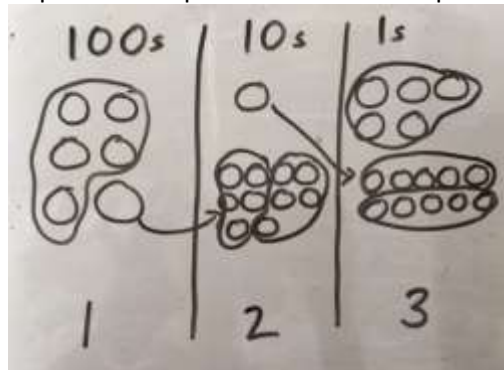


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Repeat with remainders e.g.  $643 \div 5 =$

Year 5 extend to ThHTU ÷ U

Represent the place value counters pictorially.



Children to write the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

$$\begin{array}{r} 123 \text{ r } 3 \\ 5 \overline{) 615} \end{array}$$

# Division

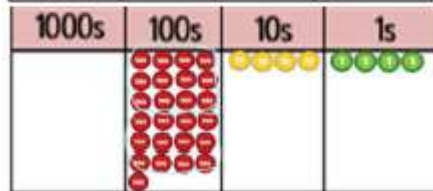
Year 6

**HTU ÷ TU Long division**  
Using place value counters

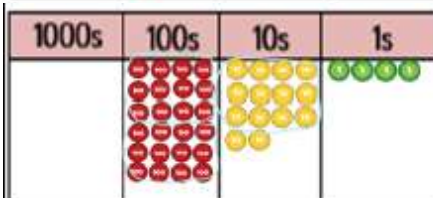
$$2544 \div 12$$



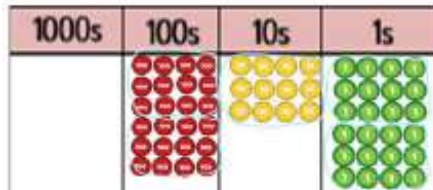
We can't group 2 thousands into groups of 12, so we exchange them for 20 hundred counters



2 groups of 12, which leaves 1 hundred. We exchange this for 10 ten counters



1 group of 12 in the tens column leaves 2 tens. We exchange them for 20 ones



Group the 24 ones into 2 groups of 12, with no remainder

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

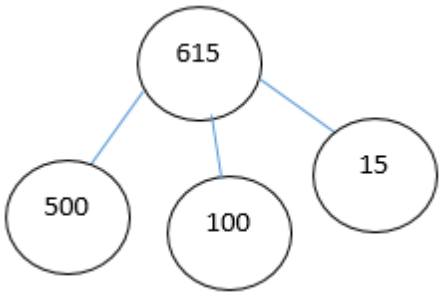
$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

# Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\underline{\quad} = 615 \div 5$$

What is the calculation?  
What is the answer?

